

FINAL EXAMINATION

ECE 627

June 9, 2009, 2-3:50 pm

KEAR 305

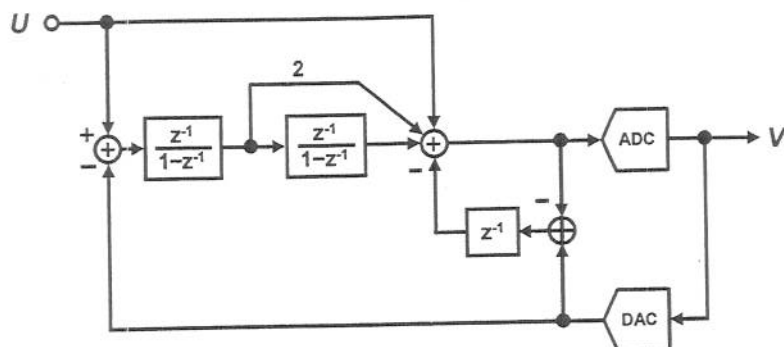
Open book, open notes

1. An $M+1$ level DAC contains M unit current sources. The i^{th} source has a value $I + dI_i$, where I is the ideal value, and dI_i is the error.

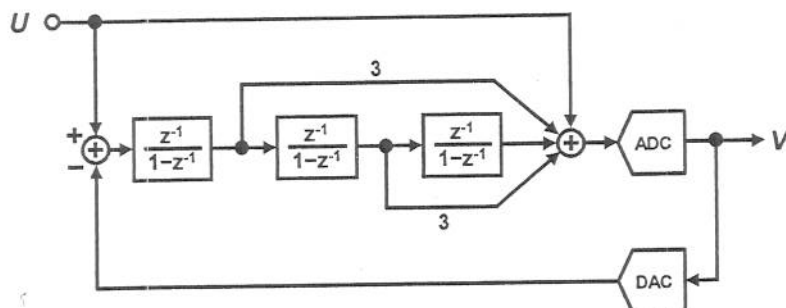
a. Find the expression for the end-point matched INL_n in terms of the dI_i when n sources are being used.

b. Find the mean-square value (MSV) of INL_n when the dI_i are uncorrelated, and the MSVs of all errors are equal to dI^2 . For what value of n is the mean-square conversion error the largest? How large is it?

2. Find the NTF and STF of the ADC shown below.



3. The input of the delta-sigma ADC shown below satisfies $-1 \text{ V} < u(n) < 1 \text{ V}$; the LSB voltage of the quantizer is 0.4 V . What is the output voltage range of the first integrator?



Solutions.

1. An $M+1$ level DAC contains M unit current sources. The i^{th} source has a value $I + dI_i$, where I is the ideal value, and dI_i is the error.

a. Find the expression for the end-point matched INL_n in terms of the dI_i when n sources are being used.

b. Find the mean-square value (MSV) of INL_n when the dI_i are uncorrelated, and the MSVs of all errors are equal to dI^2 . For what value of n is the mean-square conversion error the largest? How large is it?

a. Actual output:

$$A(n) = \sum_{i=1}^n (I + dI_i) = nI + \sum_{i=1}^n dI_i$$

End-point ideal output:

$$A_e(n) = \frac{n}{M} \sum_{i=1}^M (I + dI_i) = nI + \frac{n}{M} \sum_{i=1}^M dI_i$$

$$INL_n = \sum_{i=1}^n dI_i - \frac{n}{M} \sum_{i=1}^M dI_i = \left(1 - \frac{n}{M}\right) \sum_{i=1}^n dI_i - \frac{n}{M} \sum_{i=n+1}^M dI_i$$

$$\begin{aligned} \text{b. } \overline{INL_n^2} &= \left[n \left(1 - \frac{n}{M}\right)^2 + (M-n) \frac{n^2}{M^2} \right] dI^2 \\ &= \frac{M-n}{M^2} \left[n(M-n) + n^2 \right] dI^2 = \frac{n(M-n)}{M} dI^2 \end{aligned}$$

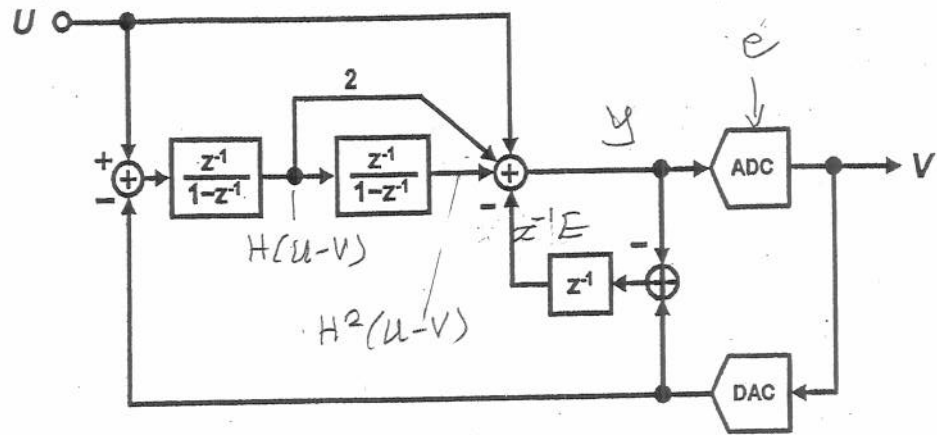
$$\frac{\partial \overline{INL_n^2}}{\partial n} = 0 \rightarrow n_{\max} = M/2 \quad (\text{or } M/2 \pm 1/2)$$

$$\overline{INL_{\max}^2} = M dI^2 / 4, \quad \text{Relative MS error } \epsilon^2 = \frac{dI^2}{I^2}$$

FS output power $M^2 I^2$ Relative MS error

$$\frac{\overline{INL_{\max}^2}}{A_{\text{FS}}^2} = \frac{(M/4) \epsilon^2 I^2}{M^2 I^2} = \frac{\epsilon^2}{4M} \rightarrow 10 \log_{10} (\epsilon^2 / 4M) \text{ dB}$$

2.



$$Y = u + 2H(u-v) + H^2(u-v) - z^{-1}E = V - E$$

$$Y = E + (H+1)^2 u - [(H+1)^2 - 1] V - z^{-1}E$$

$$(H+1)^2 V = (H+1)^2 u + (1-z^{-1})E$$

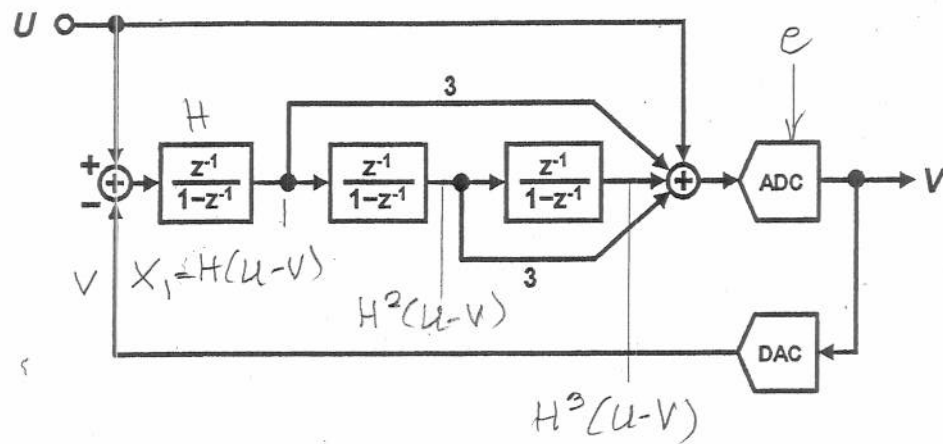
$$(H+1)^{-2} = (1-z^{-1})^2$$

$$V = u + (1-z^{-1})^3 E$$

$$\text{NTF} = (1-z^{-1})^3$$

$$\text{STF} = 1$$

3.



$$V = E + U + 3H(u-v) + 3H^2(u-v) + H^3(u-v)$$

$$V = \frac{E}{(H+1)^3} + U = U + (1-z^{-1})^3 E$$

$$H(u-v) = -z^{-1}(1-z^{-1})^2 E \rightarrow -e(n-1) + 2e(n-2) - e(n-3)$$

$$|x_1(n)| \leq 4|e|_{\max} = 4V_{\text{LSB}}/2 = 0.8V$$